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Competing for Attention in Social Communication Markets*

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Competing for Attention in Social Communication Markets

Abstract

We investigate the incentives for social communication in the new social media technologies. Three features of online social communication are represented in the model. First, new social media platforms allow for increased connectivity: i.e., they enable sending messages to many more receivers, for the same fixed cost, when compared to traditional word-of-mouth. Second, users contribute content because they derive status/image based utility from being listened to by their peers. Finally, we capture the role of social differentiation, or how social distance between people affects their preferences for messages. In the model agents endogenously decide whether or not to be a sender of information and then compete for the attention of receivers. An important point of the paper is that social communication incentives diminish even as the reach or the span of communication increases. As the span of communication increases, competition between senders for receiver attention becomes more intense resulting in senders competing with greater equilibrium messaging effort. This in turn leads to lower equilibrium payoffs and the entry of fewer senders. This result provides a strategic rationale for the so-called "participation inequality" phenomenon which is a characteristic of many social media platforms. We also show that social differentiation may enhance or deter sender entry depending upon whether or not it can be endogenously influenced by senders. Finally, we examine how the underlying network structure (in terms of its density and its degree distribution) affects communication and uncover a non-monotonic pattern in that increased connectivity first increases and then reduces the entry of senders.

1 Introduction

With the growth of social networking platforms and social media web-sites, user generated content is becoming an important form of online communication. According to a recent 2013 Pew Research Center survey adult use of social networking sites in the U.S. has gone up from 8% in 2005 to 72% in 2013.[1] The emergence of new social media platforms and technologies has made online social communication widespread between individuals who are social distant from each other. Online social communication is distinct from conventional word-of-mouth in its propagation structure: Individuals can easily communicate with their entire social network, whereas traditional word-of-mouth communication involves incurring costs that are marginal to communicating with each additional individual. One important consequence of the rapid growth of social media platforms and the ease of communication provided by them is the large amount of information received by individuals. According to Lifehack.com, a firm that helps users and firms to manage social media information, the amount of information dumped on an average daily social network user is about 54K words of text (the equivalent length of an average novel) and over 7 hours of video.[2] Therefore, senders of content often have to compete intensely for the attention of receivers, raising the question of how senders' incentives to produce content are affected by these social communication trends.

This paper analyzes the incentives to be a sender or a receiver in online social networks. In order to understand these incentives, we bring together three important features that characterize the online social media communications: First, social communication through blogging, sharing online opinions, tweeting, etc. represents a different type of effort technology on the part of the contributors than that of the traditional off-line person-to-person word of mouth communication. For an individual who wishes to send messages, traditional communication implies a marginal effort for each additional recipient in addition to the fixed cost of creating and organizing the message. In contrast, for online social communication, it is typical that once the fixed effort cost of creating the message is incurred, it is substantially easier to send the message to numerous recipients in the social network. The range of receivers for a sender is a function of the sender's network on the social media platform. The number of connections an ordinary sender can have on social networks such as Twitter are in the hundreds or even the thousands, which then potentially exposes individuals to communication from others who can be socially more distant than in the off-line world.

[1] "Social Networking Fact Sheet" - available at *http://www.pewinternet.org/fact-sheets/social-networking-fact-sheet/* (accessed 3/4/2014)

[2] "Information Overload from Social Media," - available at *http://socialmediatoday.com/socialbarrel/1637856/information-overload-social-media-infographic* (accessed 3/4/2014)

A second feature captured in our analysis is that individuals in the major online social networks take costly efforts to create and send content despite the lack of any direct monetary incentives. The main rationale for social communication that has been identified by existing research is the status based social utility that senders enjoy from reaching and being listened to by other peers in the network (Lampel and Bhalla. (2007)). A recent empirical study by Toubia and Stephen (2013) on Twitter users indicates that social image related utility is more important than intrinsic considerations in motivating users to contribute content. Accordingly, our model characterizes a sender's payoffs as a function of the number of receivers who listen to the sender. This can also be interpreted as the sender deriving utility from the amount of receiver attention.

The third feature is the role of social differentiation among agents in the network. Social media platforms allow users to connect to a network of individuals who may be heterogeneous in the extent of their social distance. The resulting social differentiation may determine how receivers decide to consume the content from a sender. For example, on Facebook information received from socially close friends may be seen as more relevant than that from socially distant contacts. This could be due to some common socioeconomic group membership. The implication for our analysis would then be that extent of social distance should govern the relevance of a sender's message for a receiver and, in turn, the likelihood that the sender will be listened to.

We tie together the features described above through an analysis of strategic competition for attention between individuals in a social communication network. The model consists of a social network of agents, each of whom has utility for social communication that they may receive from other agents. Each individual can choose to become a sender of information at some fixed cost which represents the cost of creating the necessary content for communication. Senders have utility for the expected number of receivers who will listen (pay attention) to them which reflects the idea that senders value the social/status utility arising from the extent to which they are listened to within their social network. Each sender then exerts costly effort in order to maximize the number of receivers who will pay attention to her message. Senders are able to send messages to receivers who are in their communication span or, in other words, the set of receivers with whom they can communicate. If a sender increases the frequency of sending messages, receivers are more likely to be exposed to that sender's message. Each receiver ends up getting messages from all the senders who target that receiver. A receiver's probability of listening (or paying attention) to a given sender is a function of the messaging frequency (effort) of that sender as compared to the total messaging frequency from all the senders in the receiver's targeting set. Finally, a receiver may be more or less socially distant from the sender which affects the probability with which the received message is relevant. This, in essence, sets up a framework to investigate the competition for attention between senders.

The analysis investigates the following specific questions: First, we ask how communication incentives are affected by an increased span of social communication. The competition for attention depends on senders' messaging frequency decision as they are able to reach more and more receivers. Thus, there is likely a strategic interplay between the communication span of the senders and their messaging decisions. Second, we investigate the incentive of individuals to become a sender (as opposed to a receiver) and ask how the endogenous choice of the communication span affects this incentive. Third, we examine the impact of social differentiation on communication incentives. In so far, as greater social distance implies that a sender's message would be less relevant for a receiver, we also examine how costly and endogenous efforts to increase the relevance of messages affects the competition between senders. Finally, we ask how the social network structure affects the proportion of senders in the population and examine the role of the degree distribution and network density.

Starting out our analyses with a basic model, we simplify the network structure to a regular network by placing agents on a circle where senders are connected to a continuous span of adjacent receivers on the circle. This allows us to uncover a useful insight regarding the equilibrium messaging effort (frequency) of senders. Suppose it were the case that the communication span of senders is very small such that each sender targets very few and even distinct receivers. So while each sender's reach is small, there is also little or no competition between senders for receiver attention. Consequently, senders make minimal effort in their communication. As the communication span increases, there is a direct effect through which senders get the benefit of being able to reach additional receivers. But there is also an indirect or strategic effect as senders begin to increasingly compete with each other for the attention of receivers. In equilibrium, this leads to higher messaging frequency efforts and lower sender payoffs. Consequently, when the communication span increases, a smaller proportion of the population choose to become senders, but each sender ends up sending more messages. This result can be related to what industry analysts call the participation inequality phenomenon, that even as the extent of online communication has grown only a small proportion of individuals on social media web-sites actually send content.[3]

[3]This phenomenon is also called the 90-9-1 principle meaning that typically 90% of the individuals are receivers, 9% edit or modify existing content and only 1% end up creating actual content. Recent industry reports for example show that the median tweets in the lifetime of a Twitter user is one and 10% of Twitter users produce 90% of tweets (see "New Twitter Research: Men Follow Men and Nobody Tweets," http://blogs.hbr.org/cs/2009/06/new_twitter_research_men_follo.html - accessed June 8, 2013). Other reports are even more extreme, claiming that only 0.1% of the 55 million users who have weblogs post daily.

The above result provides a strategic explanation for why participation inequality has increased in online social networks even as the costs of communication have gone down. It may also explain why some users turn away from popular social networks and are "looking for more intimate places to share items with a handful of people"[4] and why some newer social networks, such as Path are limiting the number of friends one can have[5].

We extend the model to consider the endogenous choice of the communication span which allows us to determine how the cost of targeting larger span of individuals affects the participation incentives for senders. We naturally find that as targeting people becomes less costly, senders decide to target a larger communication span. But what is interesting is senders are worse off by making this choice, even if targeting a larger span was costless. This prisoner's dilemma follows from our main results and is due to the increased competition in efforts once the communication spans are wide. Consequently, when targeting people is less costly, fewer individuals will enter and become senders, potentially explaining how communication costs reduced by social media may contribute to increasing participation inequality.

We next analyze what happens when receivers endogenously choose the amount of attention to pay to sender messages and the cost of paying attention is increasing in the total number of messages present in the environment. This characterizes a "clutter externality" which senders place on each other in competing for attention. We recover the basic point that increases in the communication span can lead to a diminution of equilibrium sender payoffs even with endogenous receiver attention. The effect of message sending costs when attention is endogenous is also interesting. Increases in the messaging costs of senders can act like a commitment for senders to not compete intensely for receiver attention which increases sender payoffs. This in turn reduces the clutter faced by receivers and induces them to pay more attention. Thus increases in the messaging costs of senders can increase receiver payoffs too. Therefore, we have the implication that higher message sending costs can not only encourage sender entry, but also make receivers better off.

Incorporating the idea of social differentiation (or the social distance between senders and potential receivers) as a measure of message relevance in the model reveals that higher social differentiation deters users from becoming senders. Surprisingly, when users can invest in making their messages more relevant to distant peers, the more difficult it is to improve message relevance, the more senders may enter the network. Intuitively, when it is harder to send relevant messages, the competition for attention becomes more localized thereby alleviating the competitive pressure on

[4]http://www.businessinsider.com/its-official-teens-are-bored-of-facebook-2013-3 - accessed Jun 8, 2013.

[5]"Why Path limits you to 150 friends"' - http://gigaom.com/2012/03/14/path-150-friends/ - accessed Jun 8, 2013.

senders. Similarly, if senders can discriminate between close and distant receivers, the competition becomes softer.

An important contribution of the paper is to model the role of the underlying social network on the communication incentives. We analyze the decision to become a sender and the ensuing communication game in an arbitrary network where, in the first stage, agents decide whether to become senders and then compete by choosing their messaging efforts. By deriving the equilibrium as a function of the network's degree distribution, we provide a general analytical solution for how the sender incentives are affected by the structure of the social network. The first point of the analysis is that receivers with intermediate degrees contribute most to sender payoffs. This is a result of two opposing effects. Receivers with many connections are more likely to be connected to many senders inducing greater sender competition. But receivers with few connections are less likely to be connected to any given sender. We find that increasing reach through the addition of links indeed results in fewer senders but only in dense enough networks where the degree distribution is sufficiently high. But we also uncover the contrary equilibrium pattern: For sparse networks we find the opposite effect, that adding links induces more agents to become senders. This highlights the interplay of two countervailing effects: On one hand, adding links to the network increases senders' reach making it more appealing to become a sender. On the other hand, adding too many links increases the competition for receivers having the opposite effect. In sum, we find a non-monotonic inverse-U shaped pattern in effect of additional links: Initially, increased connectivity attracts more senders, but after a certain point having more links hurts potential senders.

To place the main results of the paper in a more realistic setting, we analyze our model on a real-life social network. We consider a school network that captures social interaction and communication between a group of students and numerically derive the equilibrium efforts and payoff on this network. The results show similar patterns to the analytical model and support our main results. The network also allows us to study the incentives of asymmetric players to become senders. We find that the major determinant is the degree of a sender, but competition also plays an important role as it manifests in the effect of the network distance and degree of competing senders.

1.1 Related Research

Our work is related to research on competition for attention in information rich environments. In this research stream, Van Zandt (2004) proposes the idea that as the costs of sending information falls the main bottleneck is receiver attention because receivers control the allocation of attention to messages. With multiple senders there is the so-called information overload externality between senders because each sender's message crowds out the messages of other senders. In a similar vein, Anderson and de Palma (2009) investigates information congestion in a model with senders who send messages to receivers who conduct costly examination of the messages (i.e., supply attention) and shows that higher message transmission costs may increase the average benefit of a message and lead to more messages being examined.[6]

In our paper, limited receiver attention arises from the substitutability of the messages across the senders and is endogenous to a contest for receiver attention. Our analysis has two objectives which are different from the above mentioned papers: First we investigate the endogenous choice of agents to be senders or receivers which then implies that the competition for attention is linked to the entry equilibrium. This helps to characterize the entry of senders as a function of communication costs. Second, we also analyze how the network characteristics affects entry and the competition for attention.

Our paper can be related to the work on call externalities in sender-receiver interactions that are typical to telecommunication markets (Hermalin and Katz (2004), Jeon et al. (2004)). For example, Hermalin and Katz (2004) examine call externalities that are generated because the benefits enjoyed by a one side of a call is due to a message sent by the other side. They identify pricing mechanisms to restore efficiency in the calling market in the face of these externalities. Zhang and Sarvary (2014) examine externalities of user-generated content and find that users at different social media sites spontaneously differentiate through the content they post. Although our paper has some results on social differentiation, our main focus is how the incentives of users to send messages are driven by the social utility that arises from reaching and being listened to by peers in the network (Toubia and Stephen (2013), Lampel and Bhalla. (2007)). Thus the focus of our analysis is not on the pricing of call externalities, but rather on the competition for attention between senders and the manner in which it affects the incentives for sending messages.

Our model has similar objectives to a network formation game, but where we change the marginal cost of establishing additional links. An important paper in the area of endogenous network formation is Bala and Goyal (2000) who develop a model of non-cooperative network formation where individuals incur a cost of forming and maintaining links with other agents, in return for access to benefits available to these agents. Recent extensions of the model by Bramouille et al. (2004) also consider the choice of behavior in an (anti-)coordination game with network

[6] Anderson and de Palma (2012) examines consumer attention further in a model of price competition with multiple industry sectors and consumers who have limited attention (ability to process messages).

partners beyond the choice of these partners. Cabrales et al. (2011) study both network formation and endogenous production decisions[7]. The result pertaining to sender concentration in our paper is closely related in the network literature to Galeotti and Goyal (2007) who show that only a few people will invest acquiring information in a network while the majority will benefit from this information through the links. The major difference with the current paper is that we develop a rationale based on strategic communication decisions arising out of the competition for attention between senders and its effects on information transmission.

Most of the research in marketing on networks and influence relates to the question of how product adoption spreads over social links. Along these lines, Goldenberg et al. (2001) studies the diffusion process on a grid as a special type of network. First on simple, then on more complex networks the literature has uncovered several network properties and their contribution in terms of how they influence the diffusion process (Goldenberg et al. 2009, Katona et al. 2011, Yoganarasimhan 2012, Iyengar et al. 2011), and also in terms of how they help to identify influencers (Tucker 2008, Trusov et al. 2010). The focus of our paper is not on product diffusion, rather on the endogenous incentive to communicate information over a network based on the desire to compete for the attention of peers. Thus here the information in the network is a function of the strategic actions of agents.

The rest of this paper is organized as follows. In Section 2 we present our basic model, whereas in Section 3 we derive the equilibria and discuss the main results. Then, in Section 4, we model the underlying network in detail. Section 5 summarizes and provides a discussion of the main ideas of the paper.

2 The Basic Model

We start the analysis by considering a market with a unit mass of agents who are uniformly distributed on a circle of unit circumference, representing a simple, regular social network structure. Agents may send messages about some information context. All agents are potential receivers who have a given value for a sender's message conditional on their listening to the message. But each agent endogenously decides whether or not to become a sender in a manner that will be made specific below. An agent's location represents her ideal message preference or message relevance: i.e., conditional on receiving a message from a sender, the probability with which the message is relevant depends upon how close the sender of the message is to her location. Further, receivers

[7]See also Jackson and Wolinsky (1996) for an early paper concerned with the relationship between social network stability and efficiency and Jackson (2004) for a recent summary of this literature.

have limited attention and listen to only one of the senders, implying that senders have to compete with each other for receiver attention.[8]

The basic model is set up as a two-stage game sequential move game. In the first stage, each agent chooses whether or not to become a sender by incurring a fixed cost F. This fixed cost can be interpreted as the investment in obtaining the ability to send messages. This could, for example, include the cost of searching and organizing relevant information, the cost of organizing the procured information in a manner that is suitable for online delivery etc. Often these costs pertain to the time costs of searching and organizing information. While the cost of creating content for a single message might sometimes not be very high, these costs capture the potentially significant costs of senders deriving the ability to regularly send out relevant content.

Suppose that N agents choose to enter as senders symmetrically around the circle as in Salop (1979). In the second stage of the game, senders make costly efforts to attract the attention of receivers.[9] Each sender has a communication span $0 < s < 1/2$ symmetrically around her location on either side around the circle.[10] If sender i is located at x_i, s/he will be able to send messages to the interval $(x_i - s, x_i + s]$. The communication span can be seen as the network characteristic of the social media platform, with high values of s representing a greater span of social connections for any agent in the platform.[11] For their span of targeted receivers, senders must decide how much effort to put in sending messages. Let f_i denote the effort by sender i. One way to interpret f_i is the frequency of messages sent. Sender i therefore can be thought of choosing to send the message f_i times to her chosen communication span. The cost of sending a single message is assumed to be c, hence the cost of effort level f_i is $C(f_i) = cf_i$.

The probability with which a receiver listens to a sender targeting her from location i depends on (i) the number of other senders who target the receiver, (ii) the message frequencies f_j that they choose and (iii) the distance along the circumference of the receiver from the senders. From the messages that each receiver gets from all the senders that target her, she picks one randomly and considers listening to it. In other words, the probability of a receiver considering (or choosing) a message of a particular sender is proportional to that sender's relative effort compared to all other

[8]We do not need such a strong assumption but it highlights the main results. We can allow receivers to listen to multiple messages, but the key is that they have limited attention for which senders need to compete.

[9]The main results of this paper are robust to the case where agents simultaneously make decisions on entry and the messaging efforts. This analysis is presented in the Supplementary Appendix.

[10]This makes the setup similar to the models of monopolistic competition with entry that follow Dixit and Stiglitz (1977). However, apart from capturing the special aspects of social communication and attention competition, our model is distinct in that it needs to capture the non-local spatial competition for attention.

[11]This specification can also be seen as a regular network with each agent having the same number of links. In Section 4, we extend the analysis to a more general setting and model the network structure.

senders. Then, the receiver listens to the considered message with probability $1 - t \cdot \Delta x$, where $t < 2$ is the travel costs measuring how different the taste for message content is in the population and Δx measures the distance between the particular sender and receiver. The distance between agents and the taste for message content can be interpreted as the extent of social differentiation between agents. Finally, when a receiver listens to a message, the sender earns one unit of utility.

Specifically, receiver at location k listens to sender i's message with probability given by:

$$P_{ki} = \frac{f_i}{\sum_{j \in S_k} f_j}(1 - t \cdot \Delta x_{ki}), \tag{1}$$

where S_k denotes the set of senders who target receiver k. The first part of the probability formulation is the receiver's consideration or choice of a sender's message and it is in the form of a Tullock-contest function Tullock (1980)). Although proportional choice is a relatively general assumption, we can also interpret it in a manner that is compatible with social networks. If f_i be the number of messages sent by sender i during a given time period, then $\frac{f_i}{\sum_{j \in S_k} f_j}$ is the probability that receiver k listens to i's message if s/he chooses one randomly from the ones s/he receives. The second part $(1 - t \cdot \Delta x_{ki})$ is the receiver's probability of listening conditional on considering a message and t represents the "relevance" of a sender's message to the receiver or alternatively the extent of social differentiation in the market. Higher values of t imply greater social differentiation between the sender and receiver for a given distance x_{ki} which also implies lower message relevance. This in our setup implies a lower probability with which a considered message will be listened to. Given the listening probabilities, sender i's payoffs are $\pi_i = \int_{x_i - s}^{x_i + s} P_{ki} dk$.

3 Analysis

Using backward induction, consider the second stage where senders choose their effort levels. We determine the market equilibrium given that N agents had entered as senders and they simultaneously choose their efforts. To begin with, consider the case in which the communication spans are short enough, i.e., when $s < 1/(2N)$. Now there is no sender competition at the margin since each receiver can only receive messages from one sender, therefore sender efforts approach zero. The payoff of each player is thus

$$\pi^*(N, s, t) = 2 \int_0^s (1 - ty) dy = s(2 - ts) \tag{2}$$

regardless of the value of N.

When $s > 1/(2N)$, we first consider s values that make sN an integer. Then each receiver gets messages from $2sN$ senders. For the effort levels set by sender i, denoted as f_i, the second stage payoff of sender i is given by:

$$\pi_i = \frac{f_i}{f_i + \sum_{j \neq i} f_j} \cdot \int_0^s 2(1 - ty)dy - cf_i \tag{3}$$

Since we are searching for symmetric equilibria, we can posit that in equilibrium all other senders have the symmetric effort level, f_{-i}, which implies that the payoff is

$$\pi_i = \frac{f_i}{f_i + (2sN - 1)f_{-i}}(2s - ts^2) - cf_i. \tag{4}$$

The F.O.C. with respect to the effort is $\frac{(2sN-1)f_{-i}}{(f_i+(2sN-1)f_{-i})^2}(2 - ts)s = c$. Then in equilibrium $f_i = f_{-i} = f$ and so we can obtain that the second-stage choice of messaging effort and profit are:

$$f^*(N, s, t) = \frac{2sN - 1}{4csN^2}(2 - ts), \quad \pi^*(N, s, t) = \frac{2 - ts}{4sN^2}, \tag{5}$$

when sN is an integer. For values of s, that do not make sN an integer, we can show that the profits are monotone (and quadratic in s between the selected integer points), leading to the following result:

Lemma 1 *For any fixed $N \geq 2$, the equilibrium payoff $\pi^*(N, s, t)$ is increasing in s for $0 \leq s \leq 1/(2N)$ and decreasing for $1/(2N) \leq s \leq 1/2$. Furthermore, $\pi^*(N, s, t)$ is decreasing in t for any t.*

As Figure 1 shows for a fixed number of senders, each sender's payoffs initially increases and then decreases with the communication span. When the communication spans are sufficiently small, senders do not compete for the attention of receivers. Increases in the span now means that each sender can reach more receivers and thus secure higher payoffs. But as the span increases beyond $1/2N$, the targeted segments of senders begin to overlap. This results in higher degree of competition for receivers. Increasing competition induces senders to exert higher levels of equilibrium effort which, in turn, reduces payoffs. The above lemma allows us to solve the entry stage and to determine the equilibrium number of entrants in the sender market. The following Proposition characterizes the entry equilibrium:

Proposition 1 *There exists a unique equilibrium with positive entry iff $s \geq \underline{s} = \frac{1-\sqrt{1-tF}}{t}$.*

1. *As long as $s \geq \underline{s}$, the number of senders entering in equilibrium is decreasing in s, t and F.*

2. *The equilibrium sender messaging effort is increasing in s, but decreasing in t.*

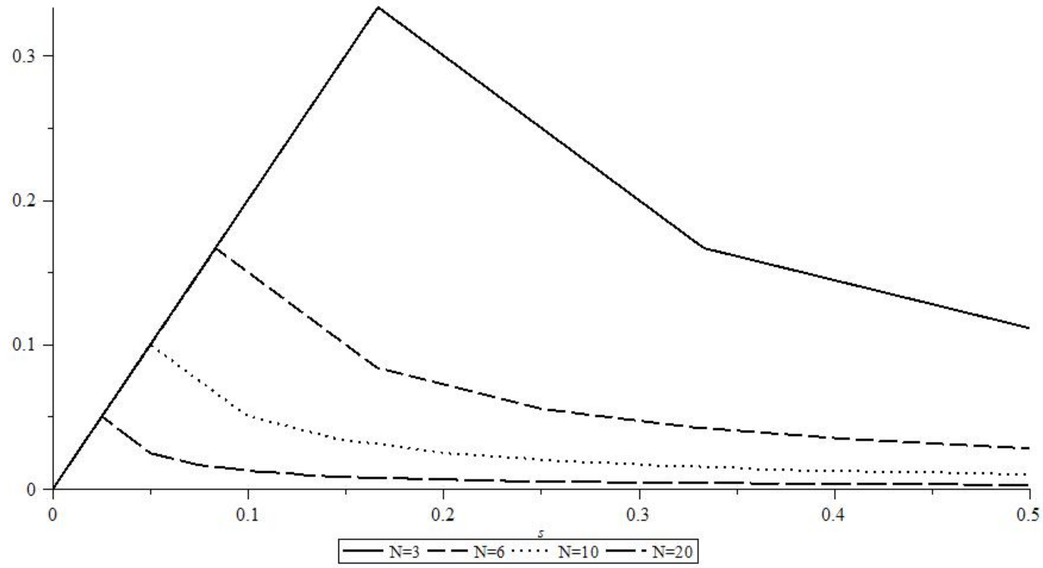
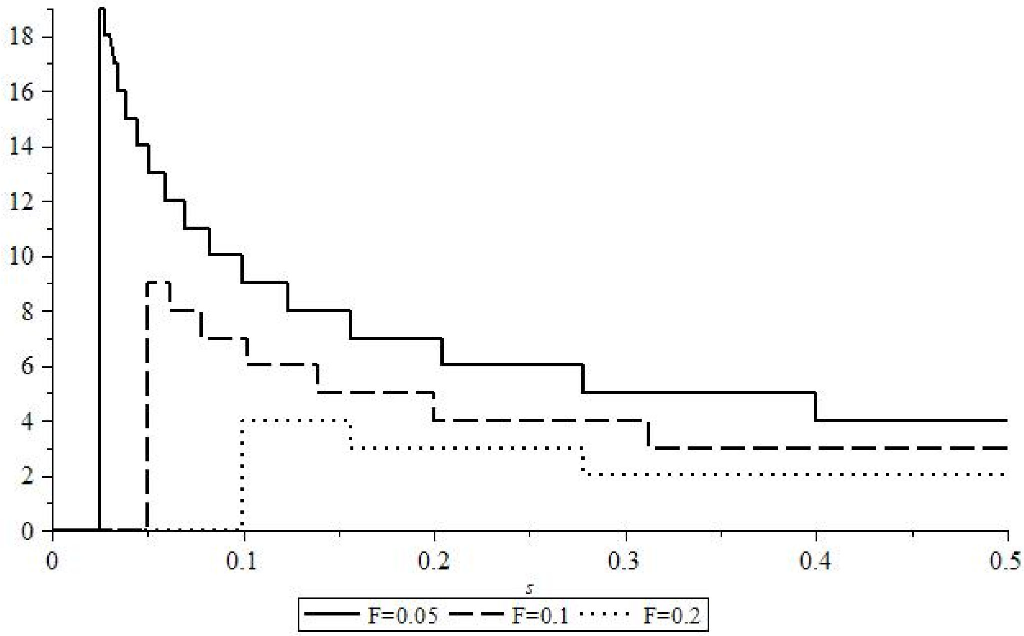


Figure 1: Equilibrium payoff as function of s for $N = 3, 6, 10, 20$ when $t = 0$.

The proposition establishes some of the basic characteristics of the competition for attention in social communication markets. The effect of the communication span on the equilibrium highlights an important point of the analysis: As the communication span increases each individual sender is able to communicate to receivers who are further away. When the span is too small there is no entry as the resulting payoffs would not cover the entry costs. Increases in the span has a positive direct effect on a sender's payoff through the greater number of receivers who are potentially reached and this can make entry more profitable. But increases in the span also creates a negative strategic effect through the increased competition between senders for receiver attention. As the span increases there is more competition and the contest for attention between senders becomes increasingly non-local. Every sender now will potentially face receivers who are targeted with a greater number of other rival senders. In equilibrium, each sender therefore ends up competing with a greater number of other senders for any receiver. Indeed, senders who enter have to respond to this increased competition by increasing their equilibrium messaging frequency. As the communication span increases fewer senders enter, but each sender ends up communicating with greater message frequency. We thus come to the interesting conclusion that an increase in the span of targeted receivers leads to greater concentration in the sender market and fewer senders in equilibrium as illustrated in Figure 2. This result is qualitatively consistent with the participation

Figure 2: Equilibrium entry as function of s for $F = 0.05, 0.1, 0.2$ when $t = 0$.



inequality phenomenon described in the introduction and provides a strategic rationale for why the proportion of individuals who contribute to online platforms continue to decrease even as it becomes easy to reach more people.

Consider next how an increase in t affects sender strategies. Like the communication span, greater social differentiation among the agents leads to fewer senders entering the market. The mechanism for this result is as follows: As t increases, for any given choice of the messaging effort, it becomes harder for a sender to win the contest for attention of a receiver who is further away. When the communication span increases, senders will end up competing for a larger set of receivers who are at greater social distance from them and so a sender finds it harder to compete for receivers who are farther away. This leads to lower sender payoffs for any given level of messaging effort and so consequently senders choose lower equilibrium levels of messaging effort. The lower payoffs also prompts fewer senders to enter. Thus in social communication markets, greater differentiation leads to fewer senders entering and hence a greater concentration in the sender market. This result can be seen as implying that the participation inequality phenomenon might be more acute for broad-based social media platforms than more specialized Internet forums or blogs which appeal to more homogenous user groups.

3.1 Endogenous Choice of Communication Span

Our analysis till this point has assumed that the communication span is a network characteristic of the social media platform. In some cases, it is possible for senders to choose efforts which determines their connectivity to others in the network. For example, users on Twitter often work hard in order to build up their followership by investing time and effort in seeking out potential followers. In this section we consider the possibility of a costly choice of the communication span by the senders. Specifically, let the basic level of the communication span be $s_l < \frac{1}{2}$, but then each sender can choose to expend effort at a cost of $z_S \geq 0$ to increase their communication span to cover the entire circle: i.e., $s_h = \frac{1}{2}$. After the entry decision, the senders who have entered simultaneously choose their communication span and the messaging frequency. The following proposition describes the second-stage equilibrium with endogenous communication span:

Proposition 2 *The symmetric equilibrium with endogenous communication span is as follows:*

1. *When* $z_S > \frac{(1-2s_l)(4-t(1+2s_l))}{16N^2 s_l^2}$, *senders choose* $s^* = s_l$, *and* $f^*_{s_l} = \frac{(2-ts_l)(2Ns_l-1)}{4N^2 k s_l}$.
2. *When* $0 \leq z_S < \frac{(1-2s_l)(4-t(1+2s_l))}{4N^2}$ *senders choose* $s^* = s_h = \frac{1}{2}$ *and* $f^*_{1/2} = \frac{(N-1)(4-t)}{4N^2 c}$.
3. *The symmetric equilibrium sender payoffs* $\pi(s^* = s_h, f^*_{s_h})$ *are strictly less than* $\pi(s^* = s_l, f^*_{s_l})$, *even when* $z_S = 0$.

This proposition characterizes the effect of the strategic choice of the communication span on sender competition. Clearly, senders expend effort and increase their equilibrium communication span when the cost z_S is sufficiently low. A smaller s_l also makes the equilibrium with $s^* = s_h$ more likely because it increases the deviation payoff of any sender who chooses the larger communication span. Further, greater social differentiation (t) reduces the incentive of senders to expend effort and so makes the equilibrium with lower sender connectivity more likely.

The third part of the proposition highlights an intriguing point: Even when the cost z_S is zero, sender equilibrium payoffs are lower than the case in which all senders are assumed to be committed to the lower communication span s_l. As z_S increases from zero, the difference in the equilibrium payoffs and those under commitment to s_l (i.e., $\pi(s^* = s_h, f^*_{s_h}) - \pi(s^* = s_l, f^*_{s_l})$) becomes even more negative. This highlights a prisoner's dilemma in the senders' choice of the communication span. When $0 \leq z_S < \frac{(1-2s_l)(4-t(1+2s_l)}{4N^2}$, senders end up choosing to increase their communication span leading to increased messaging effort competition and lower payoffs than if they were able to commit to the span level s_l. Given that the equilibrium payoffs are declining in

N, this implies fewer senders entering. A large enough z_S can actually increase sender payoffs and promote greater sender entry as it facilitates the equilibrium with the lower communication span.

The above discussion leads to the main point of this section. A sufficient increase in z_S induces senders to stay with the lower level of the communication span. This results in senders pulling back on the competition in messaging effort leading to higher equilibrium payoffs. Thus greater effort costs can act like a commitment mechanism to not increase the communication span and thereby the intensity of the messaging effort competition. Thus more senders may enter even as the costs of increasing the communication span goes up.

3.2 Endogenous Receiver Attention

In the basic model we had fixed the amount of receiver attention and focused on the competition between senders for that attention. In this section, we examine what happens when the supply of attention is costly and receivers endogenously choose the amount of attention to pay to sender messages. In doing so, we characterize the role of a "clutter externality" that senders place on each other in competing for the attention of receivers. Specifically, suppose that receivers incur costs of paying attention (listening) to sender messages which are increasing in the total number of messages present in the environment. In such a case, each sender would like to gain the attention of receivers by deploying more messages, but which will increase the attention costs faced by receivers for all messages.

To make the ideas concrete, consider the following extension to the basic set-up to account for endogenous and costly receiver attention: Post the entry decision, senders choose their messaging frequency, while all receivers simultaneously choose the amount of attention to pay in listening to messages. The attention is modeled as the probability $\gamma \in (0, 1)$ with which a receiver pays attention to the message that she randomly chooses from all the sender messages that she receives. As before receivers have unit value for the message, but the receiver only gets the value conditional on paying attention. Receiver k's attention cost is $Z_{Ak} = z_A(\sum_{j \in S_k} f_j)\gamma_k^2$. As usual the cost is increasing and convex in γ_k, but more importantly it is also increasing in the total amount of messages from all senders in the receivers targeting set S_k. This latter aspect of the attention costs represents the two-sided nature of social communication: that sender messaging efforts collectively create attention costs for receivers leading to a clutter externality, as each individual sender's messaging effort to a receiver increases the attention cost for all the other senders in the receiver's targeting set. Receiver k's payoff is a function of the amount of attention and the associated costs and can be specified as,

$$U_k = \gamma_k - z_A \left(\sum_{j \in S_k} f_j \right) \gamma_k^2. \tag{6}$$

For the analysis, we assume that the cost parameter z_A is sufficiently large so as to ensure that the equilibrium attention probability is less than one (this implies that $z_A > \frac{Nc}{(2Ns-1)(2-ts)}$).[12] The incremental sending payoffs of any sender i is given by $\pi_i = \frac{f_i}{f_i + \sum_{j \neq i} f_j} \cdot \int_0^s 2\gamma_y(1-ty)dy - cf_i$, directly generalizing the basic model. We start the analysis with the following Lemma which specifies the behavior of the equilibrium choices of senders and receivers in messaging efforts and attention, respectively:

Lemma 2 *There is a unique symmetric equilibrium in which the following holds:*

1. *The equilibrium sender messaging frequency $f^*(s, t, c, z_A)$ is increasing in s if $s < \frac{4}{4N+t}$, but is otherwise decreasing. Furthermore, f^* is decreasing in t, c and z_A.*

2. *The equilibrium receiver attention $\gamma^*(s, t, c, z_A)$ is decreasing in s and z_A, but increasing in t and c.*

This lemma shows the interplay between sender and receiver actions in the presence of the clutter externality and when receivers endogenously choose their supply of attention. When the communication span not too large ($s < \frac{4}{4N+t}$,) then consistent with the basic model increases in s leads senders to compete by increase their messaging frequency. But when $s > \frac{4}{4N+t}$ further increases in s shrinks each receiver's attention supply substantially. Senders now respond by reducing their messaging frequency. The effect of sender messaging effort costs is also noteworthy: Increases in c reduces the equilibrium messaging effort of the senders, but motivates receivers to pay more attention. On the sender side greater messaging costs essentially acts like a commitment for senders to not indulge in intense effort competition. This in turn reduces the clutter externality and induces receivers to increase their attention. Finally, the effect of social differentiation on senders is similar to the basic model and equilibrium messaging effort goes down with t. But greater social differentiation also leads to greater receiver attention. As t increases sender messages are less relevant for receivers who are farther away which makes senders reduce their messaging effort competition. This reduces the clutter faced by receivers and thus increases the attention that they pay. In other words, more specialized social media platforms might be associated with greater receiver attention.

[12] Note that if $z_A \leq \frac{Nc}{(2Ns-1)(2-ts)}$, then $\gamma^* = 1$ in which case the receiver supplies full attention, while the analysis focuses on the case where receivers supply limited attention.

Proposition 3 *In the symmetric equilibrium the sender and receiver payoffs have the following properties:*

1. *The equilibrium incremental sender payoff* $\pi^*(N, s, t, c, z_A)$ *decreases with* s*,* t *and* z_A*, and it increases with* c*.*

2. *The equilibrium receiver payoff* $U^*(N, s, t, c, z_A)$ *is decreasing in* s *and in* z_A*, but it is increasing in* t *and in* c*.*

As in the basic model, sender payoffs decrease with the the communication span. Clearly when s is small sender payoffs decrease due to the same reason as before: i.e., an increase in s leads senders to compete by increasing their messaging effort competition. When the communication span is sufficiently large ($s > \frac{4}{4N+t}$), further increases in s shrinks the supply of receiver attention to such an extent that receivers to pull back on the effort competition. Nevertheless, the greater clutter leads to lower equilibrium sender payoffs as the communication span increases.

The role of sender costs c constitutes another interesting insight of this section, in that, both sender and receiver payoffs increase even as the cost of sending messages c goes up. As discussed above greater messaging costs softens sender effort competition and increases equilibrium sender payoffs. Increases in sender messaging costs also place a lower clutter externality on receivers so that each receiver faces a market environment with fewer total messages. This increases the equilibrium supply of attention (γ^*) and thereby receiver payoffs. Thus we have the implication that higher message sending costs can not only encourage sender entry, but also increase the benefits of listening for receivers. Finally, the effect of social differentiation t on sender payoffs is similar to that in the basic model: Increases in social differentiation leads to lower messaging effort and lower sender payoffs. But as we saw in Lemma 2, greater social differentiation encourages receivers to pay more attention and therefore increases receiver listening payoffs.

3.3 Competition in Message Relevance

We now investigate the competition for attention when senders compete not only by investing in the level of the messaging effort, but also by making their messages more relevant for receivers. Recall that the probability that a receiver k listens to a sender i is given by $P_{ki} = \frac{f_i}{\sum_{j \in S_k} f_j}(1 - t \cdot \Delta x_{ki})$, where we can interpret $(1 - t \cdot \Delta x_{ki})$ as the extent of relevance of sender i's message for receiver j. In other words, the more the social distance between the sender and receiver, the less relevant is the sender's message (i.e., higher t), which implies that conditional on the message reaching the receiver, there is a lower probability that the message will be listened to.

In this section, we examine what happens if t is an endogenous costly decision for the senders along with their choice of messaging frequency. In other words, we ask what if senders could make costly efforts in order to increase the relevance of their message for the receivers? Accordingly, suppose that each sender i chooses the relevance of the message denoted by r_i at a cost of $z_R r_i^2$, in addition to the messaging frequency f_i at a cost of cf_i. The relevance decision r_i can be thought of as reducing the travel cost and is represented by $t = 2 - r_i$, and $0 < r_i \leq 2$.[13] Focusing on the values of s so that sN is an integer, we can then write the payoff given N entrants as:

$$\pi_i = \frac{f_i}{f_i + (2sN - 1)f_{-i}}(2s - (2 - r_i)s^2) - cf_i - z_R r_i^2 \tag{7}$$

The second stage equilibrium decisions in effort and relevance, can be obtained by taking the F.O.C.s w.r.t f_i and r_i. By setting $f_i = f_{-i} = f(N)$ we can derive the symmetric second stage equilibrium:

Lemma 3 *The equilibrium relevance and effort are*

$$r^*(N, s, z_R) = \frac{s}{4Nz_R}, \quad f^*(N, s, z_R) = \frac{(2Ns - 1)(s^2 + 8Nz_R(1 - s))}{16csz_R N^3}$$

The equilibrium above reveals the interaction between the two decisions. A higher cost of making the messages relevant not only reduces the extent to which senders make their messages relevant, but also their messaging frequency effort. Calculating the second stage equilibrium payoffs yields

$$\pi^*(N, s, z_R) = \frac{s^2 + 8Nz_R(1 - s) - Ns^3}{16sz_R N^3}.$$

As one would expect the equilibrium sender payoffs decrease in N and s. Surprisingly, however, sender payoffs increase even as the cost of improving message relevance z_R increases. An increase in z_R leads to lower levels of message relevance being chosen by a sender which would have a negative effect on payoffs. But there is a strategic effect following from the reduced competition that a lower relevance causes. When senders endogenously invest less in message relevance distant receivers become less valuable as they are less likely to listen. This reduces the incentive to compete with higher messaging effort. This, in turn, leads to less competition and higher payoffs. The following Proposition establishes equilibrium sender entry when message relevance is endogenous.

Proposition 4 *When s is sufficiently large, there is a unique equilibrium with $N^* > 0$ entrants. $N^*(s, z_R)$ decreases in s and increases in z_R.*

[13] This ensures that $(1 - t \cdot \Delta x_{ki}) \in (0, 1]$ and so assures the regularity of the listening probabilities P_{ki}. We also assume $z_R > \frac{s}{8N}$ to satisfy the second order condition and get an interior solution.

The main result from the previous sections continues to hold and fewer agents choose to become senders when s increases. But this proposition reveals a surprising and somewhat unexpected difference in how social differentiation influences the entry decision as compared to basic model where t was exogenous. Recall that when message relevance was an exogenous characteristic, a higher t resulted in less entry because it made it harder for a sender to compete for receivers who were in the sender's communication span but farther away. In contrast, when message relevance is an endogenous decision, higher cost of making messages relevant for distant receivers which implies lower investments in message relevance (and so can be seen as being equivalent to higher t) leads to greater number of senders in equilibrium. This is because with endogenous investments there exists the second stage strategic effect, where the higher cost of relevance induces senders to strategically pull back on the effort competition for receivers, thus increasing sender payoffs and entry. That is, when senders can invest in reducing the travel cost, more of them enter when z_R increases as investing less in message relevance allows them to soften the competition for attention.

One interpretation of the above results is the implication of social differentiation for different social media platforms. Large general interest social media web-sites are more likely interpreted as platforms where social differentiation is exogenous. As these web-sites grow and incorporate more issues and activities they will likely become more heterogenous and the implication of Proposition 1 is that this will lead to greater concentration in the sender market. In contrast, consider more specialized social web-sites where posting message content of sufficiently high relevance is necessary for entering the sender market. In these environments, the cost of message relevance may be manipulated by requiring senders to pay for or by making them incur the costs of their posts. The suggestion of Proposition 4 is then that we may actually see greater participation of senders and less skewed participation inequality in these environments.

4 Competition on a General Network

Most real-life communication is conducted through a network of links. Often an underlying social network determines conduits through which messages could be sent. The structure of these links potentially plays an important role in how senders compete for attention. In the basic model, senders were located on a circle a could reach receivers at a certain distance from them. This can be interpreted as a regular network where each person has the exact same number of connections. This simple structure allowed us to demonstrate the basic strategic forces shaping the competition for attention. In this section, we extend the analysis to a more general setting by solving the game for an arbitrary network.

Consider the network as a set of M nodes connected by links that represent the agents and the communication channels between them. To capture the network structure at a general level, we use the degree distribution: p_j denotes the proportion of nodes that have exactly j connections or degrees. In the first stage, N of the M nodes become senders, and in the second stage they set their messaging effort (frequency), sending messages to every other node that they are connected to. As in equation (1) of the basic model, we assume that receivers listen to senders with probability proportional to the messaging effort. For the sake of parsimony, we assume $t = 0$ in this section.[14] We obtain the equilibrium analytical solution given that agents know the degree distribution which is common knowledge, but they do not know their position in the network. While the latter is a reasonable assumption for sufficiently large networks, in section 4.1 we relax this assumption and consider the case where senders are aware of their exact position and that of their network neighbors.

In order to describe the results in a closed form, we introduce the notation

$$\beta(j,\alpha) = \sum_{k=1}^{j} \frac{1}{k} \binom{j}{k} \alpha^k (1-\alpha)^{j-k}$$

for the first negative moment of a Binomial distribution with (j, α) parameters. In other words, if the random variable $X \sim Binom(j, \alpha)$, then $\beta(j, \alpha) = \mathbf{E}(1/X|X > 0)P(X > 0)$ with the following properties:

Lemma 4 *There exists a $\hat{j} = \hat{j}(\alpha)$, such that $\beta(j, \alpha)$ is increasing for $1 \leq j \leq \hat{j}(\alpha)$ and decreasing for $\hat{j}(\alpha) \leq j$.*

The next proposition describes the equilibrium profits when the number of entrants in the sender market, N, is given.

Proposition 5 *If there are N senders in a network with M nodes and degree distribution (p_j), then the symmetric equilibrium sender payoffs are given by:*

$$\pi^*(N) = \frac{M-1}{N} \sum_{j=1}^{M} p_j \beta(j, N/M).$$

[14] Incorporating $t > 0$ in the network model is possible in several ways. The complexity of the analysis depends on the specific assumptions regarding the relationship between the network structure and social distance between senders. For example, when the social distance is equal to the network shortest path distance, then the results of Section 3 will directly carry over. There are also other ways to incorporate taste differences, possibly in ways that the network distance is not directly related to the differences in tastes. For example, one could place nodes in an Euclidian space to measure preferences and construct a network of social connections with some links between distant nodes.

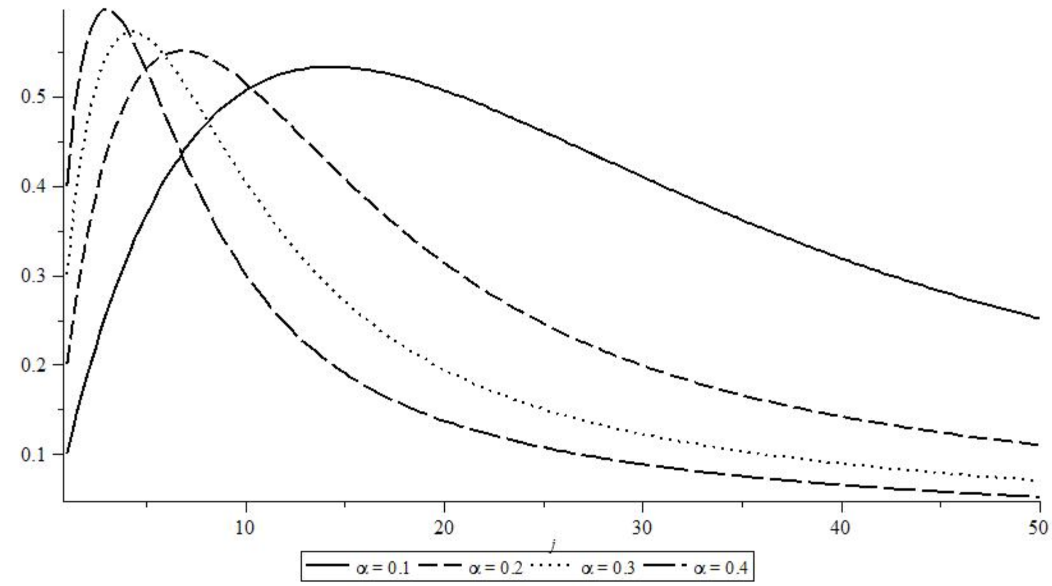


Figure 3: $\beta(j,\alpha)$ as function of j for $\alpha = 0.05, 0.1, 0.2$.

The proposition shows that the second-stage equilibrium can be fully described using only the degree distribution. Each node with a certain degree j contributes $\frac{M-1}{N}\beta(j, N/M)$ to the payoff. Thus, we can understand the role of network structure by plotting the $\beta(j, N/M)$ function fixing its second variable. Figure 3 depicts that, as shown in Lemma 4, the function is first increasing and then decreasing in j. This shape implies that nodes with an intermediate number of connections ($\hat{j}(N/M)$) contribute the most to payoffs from sending, whereas nodes with few or many links contribute less. This is a result of two opposing forces. Receivers with more connections are more likely to be connected to many senders inducing competition between senders and high sending efforts. On the other hand, receivers that have few connections are more likely to be not connected to any senders and thus not contributing to sender payoffs. As Figure 3 shows, the degree that results in the highest profits becomes lower when there are more senders, since the competitive force becomes stronger with a higher proportion of senders.

The main question of the paper is how the change in social connectivity affects the incentives to send messages. In our basic model, we used a simple parameter, the communication span s on the circle, to measure the reach of each agent. In this more general network setting the density of social connections is proportional to the number of links since the nodes are fixed. To examine the effects of increasing social connectivity we study how sender incentives change if links are added to the network.

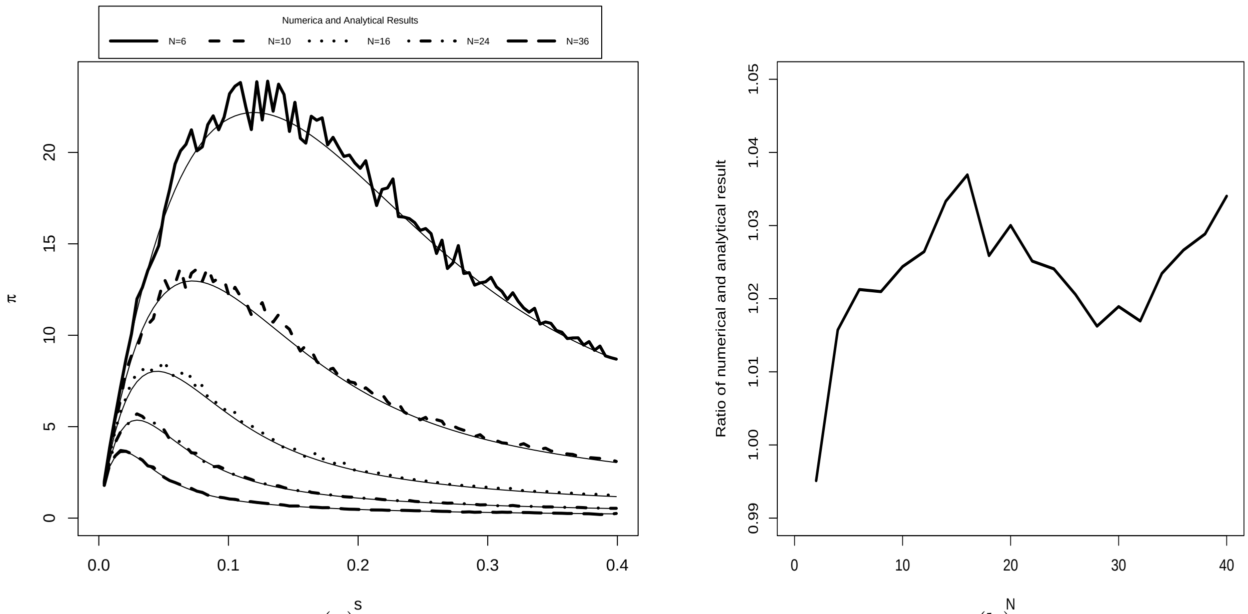
Proposition 6 *Adding links to the network increases sender entry when links are added between low-degree nodes. Adding links between high-degree nodes decreases sender entry.*

The proposition confirms an important trade-off in the competition for attention: Adding a link to the network can either increase or decrease the number of agents that decide to become senders depending on where the link is added. When links are added between already highly connected nodes sender entry decreases. As in our basic model where increasing the communication span led to more competition, such new links are likely to reduce the second stage sender payoffs because they exacerbate the negative strategic effect of increased sender competition. In contrast, links between low-degree nodes increase sender payoffs in the second stage as they increase the positive direct effect by providing more reach to senders without inducing too much competition.

An important consequence of this proposition is that if one starts from a very sparse network and adds links gradually until the network becomes very dense, then the equilibrium number of senders will follow an inverse-U shape pattern. The exact function will depend on the order in which we add the links, but in a typical process with a random component, it is likely that in the beginning links are added between relatively low-degree nodes. This would increase sender entry. But once the network becomes sufficiently dense, the nodes' degrees will increase and new links will decrease sender entry. In the next subsections, we conduct a numerical analysis based on several specific network structures to explore this question in more detail.

4.1 Regular Networks

To get additional perspective on how senders behave on different networks, we numerically analyze the equilibria on several specific networks and compare them with the general theoretical results. Apart from evaluating the main analytical results of the paper, this also allows us to check whether the assumption on senders not knowing their position in the analytical model has an impact of the results. We assume that each sender knows her exact location and is aware of her network neighbors. We start with a network that is a discrete version of our basic, circular city model of Section 3 and place M nodes on a circle and connect each node to d nodes in both directions. This is a direct generalization of the basic model with $s = d/M$. We select N random senders and numerically calculate the equilibrium sender effort levels, using an iterative approach, each player consecutively best responding until the change in actions falls below a certain threshold. Note that the random selection of senders is somewhat different from the approach in Section 3, where we



(a) (b)

Figure 4: Panel (a) shows the numerical mean payoffs of $N = 6, 10, 16, 24, 36$ randomly selected senders as a function of s with analytical prediction in the background as thin solid lines. Panel (b) show the mean ratio of numerical results and analytical predictions as a function of N.

selected equidistant senders. However, we expect the results to be very close to what we obtained in Proposition 5. Since the network is regular (all degrees are the same), the predicted mean payoffs are simply

$$\pi^*(N, d) = \frac{M-1}{N}\beta(d, N/M).$$

We ran the calculations for $M = 238$ with N values ranging from 2 to 40 with a step of 2. For the values of d we used every integer from 1 to 95, corresponding to s values ranging from 0.004 to 0.4.[15] For each of these parameter value combinations we conducted 20 runs with a different set of N random senders in each run. We took the mean of the payoffs in the 20 runs and compared with $\pi^*(N, d)$ for each N, d value pair. As shown in Figure 4, the numerical results closely match the analytical predictions. The left hand side shows the mean payoffs for different values of N with the analytical predictions as thin solid lines. On the right hand side, we depict the mean ratio of the numerical and analytical payoffs as a function of N. As the figure shows the ratio is close to 1

[15]We use the s parameter in our graphs for better comparison with the basic model. We avoided s values close to 0.5 due to long computational times. We chose the value of M to match the real-life network that we subsequently use in Section 4.2.

Table 1: Payoff regressed on parameters

	Coefficient	t**-value**
Intercept	1.11^{*}	2.41
s	36.98^{***}	6.93
s^2	-90.44^{***}	-6.94
$1/N^2$	224.21^{***}	81.87
$^{*}p < .05$, $^{***}p < .001$		$R^2 = 0.78.$

with a range of $[0.995, 1.037]$ and an overall mean of 1.023, which once again displays a close match with the analytical predictions. Thus in the specific regular network investigated, the knowledge of senders about their position does not qualitatively change the main results.

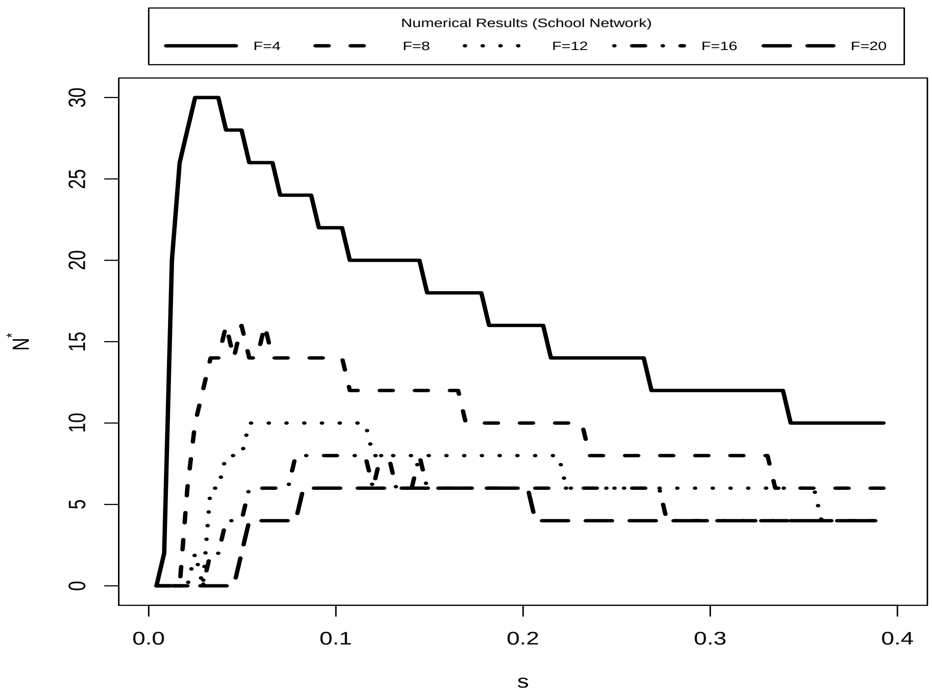
4.2 Real-Life Networks

We next examine if the main results hold for a real-life network. To do this we use the data collected by Stehlé et al. (2011) on face-to-face contact between students and teachers in a primary school. The network contains 242 nodes and 8316 undirected links, which corresponds to $s \approx 0.14$. Each link is weighted measuring the amount of time pairs of student spent in close proximity to each other. In order to examine how a change in the density affects profits we add and delete links from the network to obtain a range of s values between 0 and 0.4. To reduce density, we take advantage of the link weights and drop links sequentally, starting with the smallest weights and going up. To increase density, we use the widely adopted preferential attachment model (Albert and Barabási 2002), in which, the network grows through randomly added links. The end nodes of a new link added are selected randomly with probability proportional to the degrees of the nodes. This random network generation model has been shown to be highly representative of real-life networks. The combination of these two methods allows us to generate networks with different number of links that gives a realistic representation of how the network would grow more or less dense.

Again, we find an inverse-U shaped pattern for payoffs as a function of density (s) and the payoffs are decreasing in N. To confirm this beyond visually inspecting the results, we regressed the payoff on s, s^2, and $1/N^2$. The coefficients shown in 1 are in accordance with our expectations: s has a positive, whereas s^2 has a negative coefficient, confirming the inverse-U shape, whereas $1/N^2$ has a positive coefficient.

Calculating the second stage payoffs above allows us to examine the entry stage of the

Figure 5: Equilibrium entry as function of s for $F = 4, 8, 12, 16, 20$.



game. We assume random sender entry and solve for the highest number of entrants that can maintain a positive profit after paying the fixed cost F of entry. Since the payoffs are decreasing in N, there is a unique solution that we can calculate for any value of s and F. Figure 5 shows the number of entrants as a function of s for different fixed costs. The patterns show a clear increase for small s values and a decrease for large s values as the network gets denser. This is consistent with Proposition 5 as the two counteracting effects of competition and increased reach lead to an inverse-U shape as a function of network density. When the network is sparse the latter force dominates and additional links in the network promote more entry as senders will be able to reach more receivers, some of whom will likely be uncontested. As the network becomes denser, additional links will connect senders with receivers that are likely connected to many other senders, increasing the competition and therefore requiring higher efforts from senders. Thus entry becomes less appealing for senders when the network becomes very dense. This result is a direct generalization of what is shown in Figure 2. While the basic model could not capture increasing entry for low s values, the richer network-based model gives us the full, inverse-U shaped pattern.

It is also suggested in the figure that the value of s that maximizes entry is increasing in F. This is also consistent with our previous results: as shown in Figure 3, the $\beta(j, \alpha)$ function is

Table 2: Sender payoff regressed on network characteristics

	Coefficient	t**-value**
Intercept	9.48***	54.30
DEGREE	0.323***	1769.61
MEAN SENDER DEGREE	-0.383***	-417.46
AVERAGE DISTANCE	3.969**	50.78
$^{**}p < .01$, $^{***}p < .001$		$R^2 = 0.76$.

maximized by a lower degree as α increases. This implies that with more senders, the competitive effect starts to dominate at a lower density in the network. Since a higher fixed cost leads to fewer senders entering in general, the competitive effect starts reducing payoffs and thus sender entry at a higher network density.

4.3 Asymmetric Senders

Throughout this section, we assumed that senders are selected randomly as potential entrants do not know their position in the network. However, it is worthwhile to investigate how the position of each agent affects their payoff if they decide to become senders. In all previous numerical analysis we calculated the mean payoff of all senders. Here, we calculate each sender's payoff separately and regress it on specific network characteristics. Most importantly, we expect that a sender with a high number of connections (DEGREE) will earn a higher payoff as they are able to reach more receivers. We also calculate the *mean degree of senders* (MEAN SENDER DEGREE) as this may affect how much competition a sender may expect. Finally, a clear indication of competition is how distant senders are in the network. Therefore, we calculate the average distance (AVERAGE DISTANCE) between all senders in the network.

We ran the regression, using data obtained by numerically calculating sender payoffs in the real-life network described above. We conducted $100,000$ runs with randomly selecting 10 senders in each run. These runs provide variety in sender characteristics, allowing us to identify how the different network characteristics affect payoffs. As Table 2 shows, the results show that the position of senders matters significantly in determining their payoff and the above characteristics explain 76% of the variation. As we expected, the payoff is increasing in the degree of a sender pointing out the importance of the sender reach effect. The other two variables capture the extent of competition between senders and all suggest that more competition leads to lower payoffs. The more connections other senders have and the closer they are in the network the lower the payoff of a given sender.

5 Discussion

Social communication on the Internet is becoming a pervasive phenomenon due to the rapid growth of a variety of social media and networking platforms. On these platforms users can choose to be either producers or consumers of useful information. Some important features which define user generated content on social media are captured in this paper. The first is the status based motive for users to make costly effort and become senders of information. In our analysis this is characterized by senders deriving utility from the number of receivers who listen to them. Second, we analyze how the reduced costs of sending messages which facilitates sending messages to greater span of receivers affects communication incentives. The competition for attention in this paper then arises out of the fact that receiver value for listening to different senders is substitutable. Finally, we capture the role of social differentiation among the agents in a network. The model incorporates these features and describes the endogenous incentives of users to become senders as governed by the subsequent competition for attention of potential receivers.

On the matter of the impact of increased communication span on message sending incentives, we provide a strategic rationale for why increased connectivity stemming from new technologies in social networking and social media results in more intense competition between senders. Even as the communication span increases and it becomes easier for senders to reach more receivers, fewer senders enter and those who do compete with even higher levels of messaging effort. This result is consistent with the so called participation inequality phenomenon on the Internet. The results of the paper are also consistent with recent trends in social media where users get tired on mainstream social networks and favor smaller and more intimate venues. Industry developments also point in this direction wherein some social networks are limiting the number of connections that each user may have. We highlight the mechanism that can alleviate competition and point out that Facebook's algorithmic message filtering serves this purpose. We also analyze the role of endogenous receiver attention and the resulting clutter externality which senders place on each other. This analysis indicates an interesting role for message sending costs: Increases in message sending costs can act as a commitment for senders to soften the competition for attention and also increase receiver payoffs at the same time. Higher message sending costs can therefore not only encourage sender entry, but also make receivers better off.

We have also examined in an extension what would happen if senders are able to discriminate and target their messaging efforts based upon the social distance of the receivers from them (see the Appendix). Suppose that the receivers on a circle are divided into K equal segments and that senders are able to target these segments separately. Since senders are able to discriminate between receivers based upon their social distance, they can adjust and exert lower effort on receivers who are farther away. This allows them to mitigate the competition caused by the increased span and to command a higher payoffs compared to when they do not have the ability to target. As the analysis in the Supplementary Appendix shows we replicate our main results in that an increased communication span leads to lower profits. This implies that a higher span still decreases entry of senders, but at a lower rate than without the ability to target. Hence, we decompose the negative strategic effects of competition into an *effect due to increased reach* and an *effect due to the inability to discriminate*.[16]

Some additional extensions to the analysis presented here may be useful in providing a more nuanced understanding of communication in social media platforms. Consider the role of potential heterogeneity in messaging ability of users. In particular, suppose in the basic analysis of Section 3 of the paper, the agents in the population were heterogeneous and had high or low costs ($c+\delta$ and $c-\delta$, respectively) for sending a message. Then for a given mean cost it is possible that greater heterogeneity in messaging costs (i.e., greater δ), would lead to entry by a greater proportion of low cost (high messaging ability) senders, and an even fewer number of senders who enter compared to basic model with no cost heterogeneity. In other words, the participation inequality is likely to be even more skewed with high ability agents deterring the entry of the ones with lower ability.

But a more general problem which may be investigated in future research is the one with heterogeneity across agents in different components of costs, such as targeting span costs, and message creation costs. One may expect that if only targeting span costs differ, people with a lower cost would be more likely to become senders, but the number of senders would still decline as costs go down. However, if individuals are heterogenous in both targeting and message creation costs, it may be the case that senders with low message creation costs who would normally be more likely to enter the sender market are driven out by individuals with higher message creation, but lower targeting costs.

[16]The latter one is especially interesting in light of how Facebook is trying to mitigate it. Originally, Facebook broadcasted status updates of a user to all friends of that user. Starting in 2009, Facebook gradually updated its users' news feeds by hiding non-relevant messages. This move can be seen as consistent with facilitating targeted efforts by transmitting messages to distant users with a lower probability and by potentially reducing competition between senders by automatically targeting their efforts.

There are also information markets where senders send information which have entertainment value or consumption value (infotainment). Information may then have quality like characteristics and receivers may have greater consumption value and willingness to pay if more messages reach them. In such a case it is possible that a sender who knows that she does not face competition for receivers in her span will be motivated to send more messages. It may also be useful to consider the content of messages in detail and examine how messages sent out by senders differ, especially if the content is determined endogenously. For example, it is reasonable to assume that senders with limited resources have to make efforts in both the quality of the message and the frequency with which they send messages. Then, if the competition for attention intensifies and senders have to spend more effort on sending the same message out multiple times, it could hurt the quality of the message.

While the results in section 4.3 shed some light on how entry incentives are different across the network, we only scratch the surface here. It will be interesting to study in more detail which agents can benefit more from becoming a sender in the network. This would lead to questions pertaining to the dynamics of entry and exits of senders and the sequence in which potential senders may enter. We leave these questions to future research. Finally, in this paper we consider the structure of social connections as given. It would be interesting to examine the network formation process and see how it interacts with one's decision to become a sender or a receiver. One interesting question is whether senders are able to avoid competition by strategically forming their networks so that they target distinct sets of receivers. Overall, the economic analysis of social communication and the competition for attention in the context of new social media technologies seems to be fruitful area for future investigation.

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Appendix

Proof of Lemma 1:

Case 1: When $1/(2N) < s < 1/N$, each player exclusively sends to receivers up to a distance of $1/N - s$. Each player competes with one other player for receivers at a distance between $1/N - s$ and s. The expected payoff from the first set of receivers is $v_1 = 2\int_0^{1/N-s}(1-ty)dy = (1/N - s)(2 - t(1/N - s))$, whereas the expected payoff conditional on winning the contest from the second set of receivers is $v_2 = 2\int_{1/N-s}^{s}(1-ty)dy = (2s - 1/N)(2 - t/N)$. Thus, we can write the overall expected sender profit as

$$\pi_i = v_1 + \frac{f_i}{f_i + f_{-i}} v_2 - cf_i.$$

By taking the F.O.C., and setting $f_i = f_{-i} = f^*(N, s, t)$, we obtain the symmetric equilibrium effort $f^*(N, s, t) = v_2/4c = (2s - 1/N)(2 - t/N)/(2c)$ and the profit

$$\pi^*(N, s, t) = v_1 + v_2/2 - cf^*(N, s, t) = \frac{2N(3 - 2sN)(1 + ts) - 3t}{4N^2}.$$

The above is decreasing in both s and t.

Case 2: When $M/N < s \leq M/N + 1/(2N)$, where $1 \leq M < N/2$ is an integer, each sender competes with $2M - 1$ other senders for some receivers and $2M$ senders for other receivers. Let A denote $s - M/N$, note that $0 \leq A \leq 1/2$. Let V_{2M+1} denote the set of receivers to the right of i who are covered by i and $2M$ other senders (a total of $2M + 1$ senders). Relative to i this set is $V_{2M+1} = [0, A] \cup [1/N - A, 1/N + A] \cup .. \cup [M/N - A, M/N + A]$. Similarly $V_{2M} = (A, 1/N - A) \cup (1/N + A, 2/N - A) \cup ... \cup ((M-1)/N + A, M/N - A)$. denotes the set covered by $2M$ senders. Then the total value of the respective set of senders is $v_{2M} = 2\int_{V_{2M}}(1-ty)dy$, and $v_{2M+1} = 2\int_{V_{2M+1}}(1-ty)dy$. Thus, we can write the profit as

$$\pi_i = \frac{f_i}{f_i + (2M-1)f_{-i}} v_{2M} + \frac{f_i}{f_i + 2Mf_{-i}} v_{2M+1} - cf_i.$$

Through the F.O.C, we obtain the symmetric equilibrium effort and then the payoff

$$\pi^*(N, s, t) = \frac{v_{2M}}{(2M)^2} + \frac{v_{2M+1}}{(2M+1)^2}$$

Calculating the integrals reveals that $\pi^*(N, s, t)$ is decreasing in both s and t.

Case 3: When $M/N + 1/(2N) \leq s < (M+1)/N$, where $1 \leq M < N/2$ is an integer, we follow the same steps and obtain that

$$\pi^*(N, s, t) = \frac{v_{2M+1}}{(2M+1)^2} + \frac{v_{2M+2}}{(2M+2)^2},$$

also decreasing in both s and t. $\square$

Proof of Proposition 1: The number of entrants in equilibrium $N^* = N^*(s,t)$ is the largest integer satisfying $\pi^*(N^*, s, t) \geq F$. Examining the payoff function $\pi^*(N, s, t)$ reveals that it is strictly decreasing in N when $N \geq 1/(2s)$. However, for $1 \leq N < 1/(2s)$ the payoff $\pi^*(N, s, t)$ does not depend on N. Therefore, in order to have positive entry, the $\pi^*(1/2s, s, t) = s(2 - ts) \geq F$, yielding $\underline{s}$.

For *Part 1*, it is clear from the definition of the entry stage that N^* is decreasing in F. To see that it is decreasing in s, Lemma 1 shows that $\pi^*(N, s, t)$ is decreasing in s when $s > 1/(2N)$. We have seen above that with positive entry $s > 1/(2N^*)$, therefore we can apply the Lemma. It implies that increasing s decreases the payoffs, hence unless N is decreased the payoff falls below F. For an approximate N^* solution, we can solve $F = \frac{2-ts}{4sN^2}$ yielding $N^* = \sqrt{\frac{2-ts}{4sF}}$ for s values where sN^* is an integer. The result on t follows from the Lemma the same way.

To prove *Part 2*, examining $f^*(N, s, t)$ shows that it is increasing in s and decreasing in both N, and t. Since a higher s results in a lower N^*, the equilibrium effort, $f^*(N^*(s,t), s, t)$ is increasing in s. To see that it is decreasing in t, we write it as

$$f^*(N^*(s,t), s, t) = \frac{F}{k}\left(s\sqrt{\frac{2-ts}{sf}} - 1\right)$$

for integer values of sN^* which is clearly decreasing in t. For other values of s, the result follows similarly. $\square$

Proof of Proposition 2: The symmetric equilibrium sender payoff when all senders choose $s^* = s_l$ is $\pi(s^* = s_l, f^*(s_l)) = \frac{2-ts_l}{4N^2 s_l}$, and $f^*(s_l) = \frac{(2-ts_l)(2Ns-1)}{4N^2 c s_l}$. Any sender can deviate to $s_{dev} = \frac{1}{2}$ by incurring z_S and the optimal deviation messaging frequency and payoff for this sender will be $f_{dev}(1/2) = \frac{(4-t)(2Ns_l-1)}{16N^2 s_l^2 c}$ and $\pi_{dev}(1/2) = \frac{4-t}{16N^2 s_l^2} - z_S$, respectively. So the equilibrium when senders choose $s^* = s_l$ exists if the condition $\pi(s^* = s_l, f^*(s_l)) - \pi_{dev}(1/2) > 0$, is satisfied which implies that $z_S > \frac{(1-2s_l)(4-t(1+2s_l))}{16N^2 s_l^2}$. The condition for the existence of the symmetric equilibrium when all senders choose $s^* = \frac{1}{2}$, can be similarly established from the condition $\pi(s^* = 1/2, f^*(1/2)) - \pi_{dev}(s_l) > 0$, and it implies that $0 \leq z_S < \frac{(1-2s_l)(4-t(1+2s_l))}{4N^2}$. This proves the first part of the proposition. The second part of the proposition obtains by noting that for $z_S = 0$, $\pi(s^* = s_h, f^*_{s_h}) = \frac{4-t}{4N^2}$ and $\pi(s = s_l, f_{s_l}) = \frac{2-ts_l}{4N^2 s_l}$ and that $s_l < \frac{1}{2}$ by assumption. $\square$

Proof of Lemma 2 and Proposition 3: The incremental sending payoffs of any sender i is given by $\pi_i = \frac{f_i}{f_i + \sum_{j \neq i} f_j} \cdot \int_0^s 2\gamma_y(1 - ty)dy - cf_i$, and the payoff of receiver k is $U_k =$

$\gamma_k - z_A\left(f_i + (2sN-1)f_{-i}\right)$. Take the F.O.C's of the sender and the receiver payoffs w.r.t. f_i and γ_k, respectively. Then setting $f_i = f_{-i} = f^*$ and γ_k and the γ's in the sender F.O.C. equal to γ^* and simultaneously solving the F.O.C.'s for f^* and γ^*, we get $f^* = \frac{1}{4sN}\sqrt{\frac{(2-ts)(2Ns-1)}{z_A Nc}}$ and $\gamma^* = \sqrt{\frac{Nc}{z_A(2Ns-1)(2-ts)}}$. The relevant second order conditions are also satisfied. Substituting into the payoff functions the equilibrium sender and receiver payoffs are respectively $\pi^*(N,s,t,c,z_A) = \frac{1}{4Ns}\sqrt{\frac{(2-ts)c}{z_A N(2Ns-1)}}$ and $U^*(N,s,t,c,z_A) = \frac{1}{2}\sqrt{\frac{Nc}{z_A(2Ns-1)(2-ts)}}$. Now by taking the relevant comparative statics of the equilibrium sender and receiver actions and payoffs and noting that $s > \frac{1}{2N}$ we have the results in Lemma 2 and Proposition 3.

□

Proof of Lemma 3: As we are looking for a symmetric equilibrium, we can assume that $f_i = f_{-i}$. Then the F.O.C. with respect to r from equation (7) is $\frac{s^2}{2sN} - 2z_R r = 0$, yielding $r^* = s/4Nz_R$. Plugging $t = 2 - r^*$ into Lemma 1 provides the equilibrium effort. In order to make sure that the above is an equilibrium, we need to check the second order conditions. The requirement on the Hessian is $\frac{4s^2N(4z_R(1+2sN)(1-s)+s^2(1/(2N)+s-s^2N))}{f_i^2(1+2sN)^4} < 0$ which is satisfied when $z_R > \frac{s}{8N}$, which we have assumed in the definition of r. □

Proof of Proposition 4: Following the exact same steps as in the proof of Proposition 1, we determine how a change in the parameter values affects equilibrium entry using the second stage payoffs calculated after Lemma 3. □

Proof of Lemma 4: The proof builds on the following recursion

$$\beta(j+1,\alpha) = (1-\alpha)\beta(j,\alpha) + \frac{1-(1-\alpha)^{j+1}}{j+1}$$

that can be verified directly from the definition of $\beta(j,\alpha)$ as a sum. Examining $\beta()$ for small j values shows that $\beta(1,\alpha) = \alpha$, and $\beta(2,\alpha) = 2\alpha - (3/2)\alpha^2$, that is $\beta(2,\alpha) > \beta(1,\alpha)$ iff $\alpha < 2/3$. Let $\hat{j}(\alpha)$ denote the smallest j such that $\beta(j+1,\alpha) < \beta(j,\alpha)$. We show by induction that for any j above this value $\beta()$ is decreasing in j. Let us assume that $\beta(j+1,\alpha) < \beta(j,\alpha)$, but $\beta(j+2,\alpha) \geq \beta(j+1,\alpha)$. The recursion then implies that $(1-\alpha)\beta(j,\alpha) + \frac{1-(1-\alpha)^{j+1}}{j+1} < \beta(j,\alpha)$, that is $\frac{1-(1-\alpha)^{j+1}}{\alpha(j+1)} < \beta(j,\alpha)$ and also that $\frac{1-(1-\alpha)^{j+2}}{\alpha(j+2)} \geq \beta(j+1,\alpha)$. We can then write

$$\frac{1-(1-\alpha)^{j+2}}{\alpha(j+2)} \geq \beta(j+1,\alpha) = (1-\alpha)\beta(j,\alpha) + \frac{1-(1-\alpha)^{j+1}}{j+1} > \frac{1-(1-\alpha)^{j+1}}{\alpha(j+1)} > \frac{1-(1-\alpha)^{j+2}}{\alpha(j+2)},$$

a contradiction. Therefore, $\beta(j+2,\alpha) < \beta(j+1,\alpha)$ must hold. Similarly for all larger values of j, $\beta(.,\alpha)$ is decreasing, proving the lemma. □

Proof of Proposition 5: Since senders do not know their exact position, they only consider the number of receivers in the network, the probability that they are connected to them and the distribution of the number of other senders connected to the receivers. Let r denote a potential receiver and S_r denote the set of the receiver's neighbors that are senders. Then the payoff of sender i is $\pi_i = I(i \in S_r) \sum_{1 \le r \le M, r \ne i}^{M} \frac{f_i}{\sum_{g \in S_r} f_g} - cf_i$. Since the sender does not know its network position, s/he treats S_r as a random variable. Furthermore, as we are looking for a symmetric equilibrium, we can use the same f_{-i} effort levels exerted by all other senders. Then the expected payoff of sender i is

$$\mathbf{E}\,\pi_i = \sum_{r=1, r \ne i}^{M} \left[\sum_{j=1}^{M} p_j \sum_{k=1}^{j} \frac{k}{N} \binom{j}{k} (N/M)^k (1 - N/M)^{j-k} \frac{f_i}{f_i + (k-1) f_{-i}} \right] - cf_i$$

where j is r's number of neighbors (its distribution is p_j), $k = |S_r| \le j$, k/N is the probability sender i is connected to receiver j, and $\binom{j}{k}(N/M)^k(1-N/M)^{j-k}$ is the probability that $k = |S_r|$. Since there are $(M-1)$ receivers[17], and they are identical from the perspective of the sender, we can transform the expected profit to

$$\mathbf{E}\,\pi_i = \frac{M-1}{N} \left[\sum_{j=1}^{M} p_j \sum_{k=1}^{j} k \binom{j}{k} (N/M)^k (1 - N/M)^{j-k} \frac{f_i}{f_i + (k-1) f_{-i}} \right] - cf_i$$

Differentiating with respect to f_i term by term yields the F.O.C. Then setting $f_i = f_{-i}$ gives the equilibrium effort levels as

$$f_i^* = \frac{M-1}{Nc} \sum_{j=1}^{M} p_j \sum_{k=1}^{j} \frac{k-1}{k} \binom{j}{k} (N/M)^k (1 - N/M)^{j-k}$$

and the equilibrium payoffs as

$$\pi_i^* = \frac{M-1}{N} \sum_{j=1}^{M} p_j \sum_{k=1}^{j} \frac{1}{k} \binom{j}{k} (N/M)^k (1 - N/M)^{j-k} = \frac{M-1}{N} \sum_{j=1}^{M} p_j \beta(j, N/M)$$

□

Proof of Proposition 6: For any j, by definition $\beta(j, \alpha)$ is strictly decreasing in α, which implies that $\pi^*(N)$ is strictly decreasing in N. As before, let N^* denote the largest N such that $\pi^*(N) \ge F$. When adding a link between low-degree nodes $\pi^*(N)$ increases according to

[17] Here, we allow other senders to act as receivers to make our model consistent with the basic model. This is a minor technical assumption which does not affect the results substantially.

Proposition 5 and Lemma 4. Since $\pi^*(N)$ is decreasing in N, the equilibrium N^* increases with the addition of the new link. When adding a link between high-degree nodes the exact opposite happens. $\square$